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Fraunhofer-Institut für Bauphysik

G. Rosenhouse*)

The Application of Potters' Method for 2-D Numerical Analysis of Acoustic Fields in Wave-Guides

Introduction

The equations of motion which describe acoustic wave propagation in liquids and homogenous isotropic elastic bodies may be given in terms of scalar and both scalar and vector velocity potentials respectively. The application of finite differences schemes in those cases describes the field equations for the decoupled potentials, and the boundary conditions for which the coupling still exists [1].

Formulation

Specifically, for fluids within wave-guides, Helmholtz equations should be satisfied at each of the field modal points, whilst the boundary conditions include the following possibilities [2]:

1. Sound absorbing boundaries;
2. Elastically reacting boundaries;
3. "ϱc" boundary conditions which allow analysis of a finite domain within an infinite space;
4. Both absorbing and elastically reacting boundaries;
5. Boundaries along which there are line and point sources.

The analogy between the field equations within the fluid and absorber at the lining of the wave-guide follows [3].

Compatibility and equilibrium conditions are expected at the interface between the fluid and the absorber, which may be defined also by the following equations [2]:

*)Prof. Rosenhouse arbeitet als Gastwissenschaftler vom Technion-Israel Institute of Technology, Haifa am IBP.
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For $y = y_b$ and varying x one obtains:

$$(\alpha + i\beta)\, \varrho\, \Omega^2 \sum_{n=1}^{\infty} \left\{ \phi_n(x)\, \alpha_{dyn,n} \frac{\int_0^{l_x} p(x)\, \phi_n(x)\, dx}{\Omega_n^2 \int_0^{l_x} m\, \phi_n^2(x) dx} \right\} = i\, \underline{k}\, p(x) - (\alpha + i\beta) \frac{\delta_p}{\delta_x}\,, \qquad (1)$$

and the result for $x = x_b$ and varying y is:

$$(\alpha + i\beta)\, \varrho\, \Omega^2 \sum_{n=1}^{\infty} \left\{ \phi_n(y)\, \alpha_{dyn,n} \frac{\int_0^{l_y} p(y)\, \phi_n(y)\, dy}{\Omega_n^2 \int_0^{l_y} m\, \phi_n^2(y) dy} \right\} = i\, \underline{k}\, p(y) - (\alpha + i\beta) \frac{\delta_p}{\delta_y}\,, \qquad (2)$$

$\alpha + i\beta$ is the impedance ratio. The sum at the right hand side of equations (1) and (2) represents the vibrating modes $\Theta_M(x)$ and $\Theta_M(y)$ of the vibrating surface of the boundary. p is the resulting pressure along the boundaries.

Solution method

In many cases the resulting set of equations my be arranged in a tri-diagonal form.

A typical arrangement is depicted in figure 1. This specific form allows the use of Potters' algorithm [4].

The scheme of the solution for the set in figure 1 is given in table 1. One may find by using this method that this kind of solution saves a lot of computer memory, and that the process is very quick.

Illustration

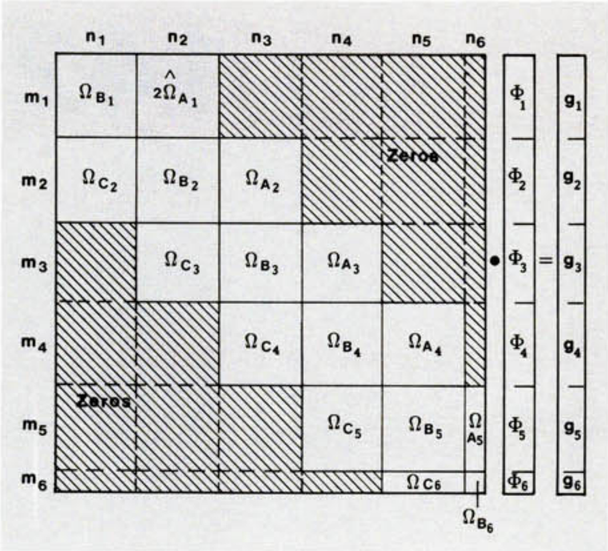

Fig. 1: Arrangement of submatrices and subvectors of a typical system, following the modified Potters' method

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i	P_i	$\bar{X}_i$	$\bar{X}_i$ $(g_1=g_2=g_3=g_4=0)$	Φ_i
1	$\Omega_{B1}^{-1}(2\hat{\Omega}_{A1})$	$\Omega_{31}^{-1} g_1$	0	$\bar{X}_1 - P_1\Phi_2$
2	$(\Omega_{B2}-\Omega_{C2}P_1)^{-1}\Omega_{A2}$	$(\Omega_{B2}-\Omega_{C2}P_1)^{-1}(g_2-\Omega_{C2}\bar{X}_1)$	0	$\bar{X}_2 - P_2\Phi_3$
3	$(\Omega_{B3}-\Omega_{C3}P_2)^{-1}\Omega_{A3}$	$(\Omega_{B3}-\Omega_{C3}P_2)^{-1}(g_3-\Omega_{C3}\bar{X}_2)$	0	$\bar{X}_3 - P_3\Phi_4$
4	$(\Omega_{B4}-\Omega_{C4}P_3)^{-1}\Omega_{A4}$	$(\Omega_{B4}-\Omega_{C4}P_3)^{-1}(g_4-\Omega_{C4}\bar{X}_3)$	0	$\bar{X}_4 - P_4\Phi_5$
5	$(\Omega_{B5}-\Omega_{C5}P_4)^{-1}\Omega_{A5}$	$(\Omega_{B5}-\Omega_{C5}P_4)^{-1}(g_5-\Omega_{C5}\bar{X}_4)$	$-(\Omega_{B5}-\Omega_{C5}P_4)^{-1}\hat{g}_5$	$\bar{X}_5 - P_5\Phi_6$
6	•	$(\Omega_{B6}-\Omega_{C6}P_5)^{-1}(g_6-\Omega_{C6}\bar{X}_5)$	$-(\Omega_{36}-\Omega_{C6}P_5)^{-1}(\hat{g}_5+\Omega_{C6}\bar{X}_5)$	$\bar{X}_6$

$\hat{g}_i = -g_i$

Table 1: A scheme which solves the problem defined in figure 1

A test problem which is described in figure 2a was divided into the grid shown in figure 2b. A representative map of isophones is depicted in figure 2c.

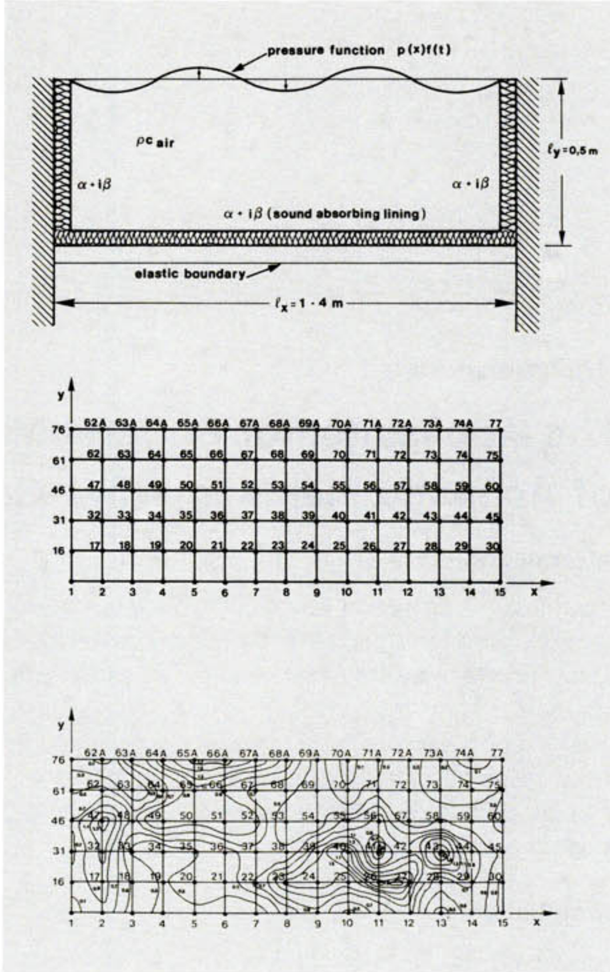

Fig. 2: Solution of a test problem

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FRAUNHOFER-INSTITUT FÜR BAUPHYSIK
7000 Stuttgart 80, Nobelstraße 12, Tel.(0711)6868-00
8150 Holzkirchen (OBB), Postf. 1180, Tel. (08024)643-0

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